Stochastic control of UAV on the basis of robust filtering of 3D natural landmarks observations

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Abstract. This work considers the tracking of the UAV (unmanned aviation vehicle) on the basis of on-board observations of natural landmarks including azimuth and elevation angles. It is assumed that either UAV’s cameras are able to capture the angular position of reference point and to measure the angles of the sight line. Such measurements involve the real position of UAV in implicit form, and therefore some of nonlinear filters such as Extended Kalman filter (EKF) or others must be used in order to implement these measurements for UAV control. Recently it was shown that modified pseudomeasurement method may be used to control UAV on the basis of the observation of reference points assigned along the UAV path in advance. However, the use of such set of points needs the cumbersome recognition procedure with the huge volume of on-board memory. The natural landmarks serving as such reference points which may be determined on-line can significantly reduce the on-board memory and the computational difficulties. The principal difference of this work is the usage of the 3D reference points coordinates which permits to guide the UAV along the path with varying altitude which is extremely important for successful performance of some autonomous missions. One more novelty of this approach is the usage of robust RANSAC taking into account the UAV motion model. The article suggests the estimation and control algorithm for tracking given reference path under external perturbation and noised angular measurements.

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Keywords: UAV, visual odometry, video stream, feature points, Kalman filter, control.

1 Introduction

Modern UAV’s navigation systems use the standard elements of INS (inertial navigation systems) along with GPS, which permit to correct the bias and improve the UAV localization which is necessary for resolving mapping issues, targeting and reconnaissance tasks [2]. The use of computer vision as secondary or primary method for autonomous navigation of UAV has been discussed frequently in recent years, since classical combination of GPS and INS systems
can not sustain autonomous flight in many situations [5]. The performing of autonomous missions usually needs so-called data fusion which is a difficult task especially for standard INS and vision equipment since the information obtained from cameras is in different form and needs the additional on-board memory and special recognition algorithms.

1.1 Visual-based navigation approaches

Several studies have demonstrated the effectiveness of approaches based on motion field estimation and feature tracking for visual odometry [6]. Vision based methods have been proposed even in the context of autonomous landing management [5]. In [7] Caballero et al. propose a work on visual odometry based on geometric homography. However, the homography analysis use only 2D references’ points coordinates, though for evaluation of the current UAV altitude the 3D coordinates is necessary. Here we develop the approach suggested in [12] to the case of 3D coordinates correspondence of the reference points observed by on-board camera and the reference points on the map loaded into UAV’s memory before the mission start. During the flight these maps are compared with the frame of the land, directly observed with the help of on-board video camera. All such approaches presumes the presence of some recognition system, in order to detect such points, nominated in advance. Examples are the special buildings, cross of roads, tops of mountains and so on. The principal difficulties are the different scale and aspect angles of observed and stored images which leads to the necessity of huge templates library in the memory of UAV control system. Here one can avoid this difficulty, because of usage another approach based on observation of so-called feature points [11] that are the scale and the aspect angle invariant. For this purpose the technology of feature points [9] is used. As a result one can detect current location and orientation without time-error accumulation. These methods are invariable to some transformations and noise-stable so that predetermined maps can vary on height, season, luminosity, weather conditions, etc. from the same terms during the flight. This technology appeared in [10]. Contribution of our work is the usage of modified unbiased pseudo measurements filter for bearing only observations of some reference points with known terrain coordinates.

1.2 Kalman filter

In order to get the metric data from visual observations one needs first to make observations from different positions (i.e. triangulation) and then to use the nonlinear filtering. However, all nonlinear filters either have unknown bias [13] or very difficult for on-board implementation like the Bayesian type estimation [3], [4]. Approaches for the position estimations based on bearing-only observations had been analyzed long ago especially for submarine applications [14] and nowadays for UAV applications [2]. Comparison of different nonlinear filters for bearing-only observations in the issue of the ground-based object localization [15]
shows that EKF (extended Kalman filter), unscented Kalman filter, particle filter and pseudomeasurement filter give almost the same level of accuracy, while the pseudomeasurement filter is usually more stable and simple for on-board implementation. This observation is in accordance with rather old results [14], where all these filters were compared in the issue of moving objects localization. It have been mentioned that all these filters have bias which makes their use in data fusion issues rather problematic [16]. The principle demand to such filters in data fusion is the non-biased estimate with known mean square characterization of the error. Among the variety of possible filters only the pseudomeasurement filter can be modified to satisfy the data fusion demands. The idea of such non-linear filtering has been developed by V. S. Pugachev and I. Sinitsyn in the form of so-called conditionally-optimal filtering [17], which provides the non-biased estimation within the class of linear filters with the minimum mean squared error. In this paper we use such filter for the UAV position estimation and give the algorithm for the path planning along with the reference trajectory under external perturbations and noisy measurements.

1.3 The paper structure and the results outline

In the next section 2 the new approach to the usage of 3D landmarks had been presented. This is principal novelty of the article. The modified method of pseudomeasurements is given in Sections 3 and 4. Our approach to this method is free of bias in the contrary to the standard one. Moreover due to the absence of bias one can obtain the estimation of quadratic errors which is important for the data fusion with INS. Section 5 describes the robust filtering of RANSAC on the basis of the model of UAV motion. In Section 6 we suggest the control algorithm which provides the tracking of the reference trajectory under the motion and measurements perturbations. Since the general problem of tracking on the basis of random measurement and with the nonlinear motion model does not have the explicit solution we use the suboptimal (locally optimal) algorithm. Finally in Section 7 we give the results of modelling.

2 Random sample consensus for isometry

On every step we are dealing with two images of 3D landscape. The first image $I_c$ obtained from the on-board UAV camera whose position is unknown and should be found. The second image $I_c'$ was taken previously from the known position and laid in UAV memory. ASIFT method is used on both images to find feature points specified in pixels

$$c_i = \begin{bmatrix} c_{xi} \\ c_{yi} \end{bmatrix}$$

and

$$c_i' = \begin{bmatrix} c_{x'i} \\ c_{y'i} \end{bmatrix}$$
and calculates their descriptors. Using obtained descriptors correspondence between images is constructed. The feature points are combined in pairs. However, many of these pairs are roughly wrong, we will call such pairs as outliers. The Cartesian coordinate system is given which is rigidly connected with the Earth. We will call it the Earth’s coordinate system. We have a height terrain map over which the flight is made, and we know from what position image \( I_2 \) was taken. Therefore, we can determine the coordinates of the points
\[
\mathbf{r}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix},
\]
which generated \( c'_i \) points in the Earth coordinate system. However, if \( i \)-th pair of points is not an outlier, then the point \( \mathbf{r}_i \) also generated a point \( c_i \) in the UAV camera. Another Cartesian coordinate system is rigidly connected with UAV camera. The axis of the camera is directed along the positive direction of the axis \( z \). Transformation from the Earth’s coordinate system to the UAV coordinate system has the form
\[
\mathbf{r}' = A(r - b),
\]
where \( b \) — coordinates of the camera in the earth coordinate system, \( A \) — orthogonal (\( AA^T = I \)) rotation matrix, defining the orientation of the UAV camera. Then the points \( \mathbf{r}_i \) in the camera coordinate system are
\[
\mathbf{r}'_i = A(\mathbf{r}_i - b).
\]
Let’s consider how the points \( \mathbf{r}_i \) generate feature points \( c_i \). We use the model of the camera obscura. This camera realizes the central projection on the plane:
\[
c_i = \begin{bmatrix} \rho_{xi} \\ \rho_{yi} \\ \rho_{zi} \end{bmatrix},
\]
where
\[
\begin{vmatrix} \rho_{xi} \\ \rho_{yi} \\ \rho_{zi} \end{vmatrix} = \rho_i = K \mathbf{r}'_i = K(\mathbf{r}_i - b),
\]
where \( K \) is known calibration matrix of the camera. Thus, the task is to assess \( A \) and \( b \) under known \( c_i, \mathbf{r}_i, K \). The minimum number of feature points pairs needed to solve this task is 3. First let’s solve the problem under the assumption that we have just 3 pairs:
\[
i = \{1, 2, 3\}.
\]
Points \( \mathbf{r}'_i \) form a triangle in the space with the following sides:
\[
\rho_1 = ||\mathbf{r}'_2 - \mathbf{r}'_3||_2, \rho_2 = ||\mathbf{r}'_3 - \mathbf{r}'_1||_2, \rho_3 = ||\mathbf{r}'_1 - \mathbf{r}'_2||_2.
\]
Meanwhile, since the rectilinear propagation of the light, each point \( r'_i \) lies on the beam \( r' = a_i t \), where \( t \) — parameter, and
\[
a_i = K^{-1} \begin{bmatrix} c_{xi} \\ c_{yi} \\ 1 \end{bmatrix}.
\]

To find \( r'_i \) we have to find 3 parameters: \( t_i, i = \{1, 2, 3\} \). The following system of quadratic equations should be solved to do this:
\[
\begin{aligned}
(a_2 t_2 - a_3 t_3)^2 (a_2 t_2 - a_3 t_3) &= \rho_1^2, \\
(a_3 t_3 - a_1 t_1) (a_3 t_3 - a_1 t_1) &= \rho_2^2, \\
(a_1 t_1 - a_2 t_2)^2 (a_1 t_1 - a_2 t_2) &= \rho_3^2.
\end{aligned}
\]

A fast way of solving of this system of equations is developed. Note that with a certain \( t_1 \) the system is solved analytically or has no solution. Parameter \( t_1 \) can be found by the bisection method. Now we know the coordinates of three points on the Earth surface in the camera coordinate system \( r'_i \). We have the same points in the Earth coordinate system \( r_i \). The connection between them is:
\[
r'_i = A (r_i - b).
\]
Matrix \( A \) is orthogonal, so if \( y = Ax \), then \( ||y||_2 = ||x||_2 \). Using this we obtain:
\[
r'^T_i r'_i = (r_i - b)^T (r_i - b).
\]
So we got rid of the \( A \) and obtained the problem of finding the intersection of three spheres which can be solved analytically. This problem may have 2 solutions, one of them will be rejected after. When \( b \) are found, solutions for \( A \) is as follows:
\[
A = |r'_1 |r'_2 |r'_3 | r_1 - b |r_2 - b |r_3 - b|^{-1}.
\]
We choose from two options only the one for which the following is right:
\[
\begin{bmatrix}
|a_{11}| & a_{12} \\
|a_{21}| & a_{22} \times a_{23} , but not \\
|a_{31}| & a_{32} \times a_{33} \\
\end{bmatrix} = -
\begin{bmatrix}
|a_{11}| & a_{12} \\
|a_{21}| & a_{22} \times a_{23} \\
|a_{31}| & a_{32} \times a_{33} \\
\end{bmatrix},
\]
which corresponds exactly to the turn. Now we found \( A \) and \( b \) using 3 pairs of feature points and height map. However, this approach alone is not suitable as the final solution due to the following problems:

1. method will give knowingly false solution or no solution at all if among 3 points there will be outliers.
2. there is a strong dependence from the noise in feature points location.

Both problems can be solved with random sample consensus (RANSAC) [18]. From the general selection of points we randomly select \( N \) times a subsample of size 3. For each subsample \( j = \{1, 2, \cdots, N\} \) we calculate \( A_j \) and \( b_j \), which allows to simulate the generation of all feature points on the UAV camera
\[
c_{ji} = \begin{bmatrix} \rho_{jx} \\ \rho_{jy} \\ \rho_{jz} \end{bmatrix}, \quad \rho_{ji} = K r'_{ji} = K A_j (r_{ji} - b_j).
\]
Then we can evaluate which points are the outliers by the threshold \( s_{ji} = 1(||c_i - c_{ji}||_2 < d) \), where \( d \) is the manually selected constant threshold. \( s_{ji} = 0 \) means that \( i \)-th pair on \( j \)-th pair is counted as an outlier, otherwise \( s_{ji} = 1 \). The choice for \( A \) and \( b \) is as follows \( A = A_J, b = b_J \) where

\[
J = \arg\max_j \sum_i s_{ji}.
\]

Thus, we are likely to solve the problem of outliers. Next we find the required number of \( N \) subsamples of 3 size such that among them will be at least one subsample without outliers with probability \( p \). Let the number of outliers will be \( 1 - w \). It’s easy to see that

\[
N(p) = \frac{\log(1 - p)}{\log(1 - w^3)}.
\]

For example, in the case when \( w = \frac{1}{2} \): \( N(0.9999) \approx 69 \), which shows the high efficiency of algorithm. Note that knowledge of the values \( w \) is not necessary for the algorithm. After that points marked as outliers are removed from consideration. The clarification of the response is made by the numerical solution of the following optimization problem on the remaining points:

\[
\{A^*, b^*\} = \arg\min_{A, b} \sum_i ||c_i - c_i(A, b)||_2^2.
\]

We make the assumption that the noise has a Gaussian, so the second problem of noise reduction is solved. The result of RANSAC for ISOMETRY performance is shown in Fig. 1.

3 Filtering problem statement

The problem of the bearing-only filtering is considered to determine the coordinates of UAV which can observe some objects with precisely known coordinates. These object can be either the set of well recognizable objects or the network of radio-beacon stations with well specified frequency and known coordinates. In this work the role of beacons play the feature points determined with the aid of RANSAC algorithm. The UAV has the standard set of INS devices which enables to perform the flight with some degree of accuracy which, however, is not enough for the mission completion. The UAV motion is defined by the angle between the projection of velocity to the plane \( y0x \) and the axis \( OX - \theta(t) \) and the angle between the vector of velocity and the plane \( y0x - \gamma(t) \) [22].

3.1 Model of the UAV motion

We assume 3D UAV motion described by the following equations

\[
X(t_{k+1}) = X(t_k) + VP(t_k)\Delta t + W(t_k), \quad (1)
\]
Fig. 1. Left - image $I_c$ of on-board camera of UAV, right - template image loaded in UAV memory in advance. Right lines show the correspondence between singular points of ASIFT type after rejection of outliers with the aid of RANSAC for ISOMETRY. Spurious difference between left and right landscapes results from different observation seasons, scale and aspect angle. Anyway, the correspondence between the reference points, for example, between two lakes is evident.

where $t_k = k \Delta t$, $k = 1, 2, \ldots$ and

$$X(t_k) = (X(t_k), Y(t_k), Z(t_k))^T$$

is the vector of state, $V$ – UAV’s velocity,

$$P(t_k) = \begin{pmatrix}
\cos \gamma(t_k) \cos \theta(t_k) \\
\cos \gamma(t_k) \sin \theta(t_k) \\
\sin \gamma(t_k)
\end{pmatrix},$$

$$W(t_k) = (W_x(t_k), W_y(t_k), W_z(t_k))^T$$

is the vector of current perturbations, modelling the turbulence component of the wind acting along axis $(OX, OY, OZ)$, having zero means and variances $(\sigma^2_x, \sigma^2_y, \sigma^2_z)$, correspondingly; The controls $\theta(t_k)$ and $\gamma(t_k)$ are the angles between projection of vector-velocity on the $y0x$ plane and the axis $OX$, and between vector of velocity and the axis $OX$, correspondingly, these angles define the nominal UAV motion.

3.2 Measurements

Assume that $X_i = (X_i, Y_i, Z_i)$ are the coordinates of $i$-th object and $\phi_i(t_k), \lambda_i(t_k)$ are the bearing angles on that object. At moment $t_k$ these angles satisfy the relations:

$$I(t_k) \frac{Y_i - Y(t_k)}{X_i - X(t_k)} = I(t_k)(\tan \phi(t_k) + \varepsilon^\phi_k)$$

$$I(t_k) \frac{Z_i - Z(t_k)}{Y_i - Y(t_k)} \sin \phi(t_k) = I(t_k)(\tan \lambda_i(t_k) + \varepsilon^\lambda_k),$$

(2)
where one can assume that \( \varepsilon_k^\phi \sim \mathcal{N}(0, \sigma_\phi^2) \), \( \varepsilon_k^\lambda \sim \mathcal{N}(0, \sigma_\lambda^2) \) are uncorrelated random variables with zero means and variances \( \sigma_\phi^2, \sigma_\lambda^2 \), defined as errors in measurement of tangents of angles \( \phi_i(t_k), \lambda_i(t_k) \), and forming the white noise sequences [22]. \( I(t_k) \) is an indicator function which is equal to 1 if at time \( t_k \) UAV is making measurements.

4 Modified method of pseudomeasurements

4.1 Linear measurements model

The idea of the pseudomeasurement method is to separate in (2) the observable and non observable values, which gives the following measurement vector [19], [20]:

\[
m_k = \begin{pmatrix} m_k^\phi \\ m_k^\lambda \end{pmatrix} = \begin{pmatrix} Y(t_k) \cos \phi_i(t_k) - X(t_k) \sin \phi_i(t_k) \\ + \varepsilon_k^\phi (X_i - X(t_k)) \cos \phi_i(t_k) \\ Z(t_k) \sin \phi_i(t_k) \cos \lambda_i(t_k) - Y(t_k) \sin \lambda_i(t_k) \\ + \varepsilon_k^\lambda (Y_i - Y(t_k)) \cos \lambda_i(t_k) \end{pmatrix}.
\]

Thereby we obtain the system (3) of linear measurement equations, though the noise variance depends on unobservable coordinates. By using V. S. Pugachev method [17] one can obtain the unbiased estimation and the variance evaluation with the aid of prediction-correction filter [19], [20], [21]. Details of the filtering and controls are given in [22].

4.2 Prediction-correction estimation

Assume that at the moment \( t_k \) we have unbiased estimation \( \hat{X}(t_k) = (\hat{X}(t_k), \hat{Y}(t_k), \hat{Z}(t_k)) \) such that

\[
E(\hat{X}(t_k)) = X(t_k)
\]

with the following matrix of the mean-square errors

\[
\hat{P}(t_k) = \begin{pmatrix} \hat{P}^{xx}(t_k) & \hat{P}^{xy}(t_k) & \hat{P}^{xz}(t_k) \\ \hat{P}^{yx}(t_k) & \hat{P}^{yy}(t_k) & \hat{P}^{yz}(t_k) \\ \hat{P}^{zx}(t_k) & \hat{P}^{zy}(t_k) & \hat{P}^{zz}(t_k) \end{pmatrix}
\]

In practice, the initial time the state is known.

Problem 1. Get the estimation of the UAV position at time \( t_{k+1} \) on the basis of previous estimations \( \hat{X}(t_k), \hat{P}(t_k) \), observations \( \phi_i(t_{k+1}), \lambda_i(t_{k+1}) \), known position of \( i \)-th observable object \( X_i \), and known parameters of the motion equations (1) in the interval \( [t_k, t_{k+1}] \). In other words one needs to find the unbiased estimations of \( \hat{X}(t_{k+1}) \) and the matrix \( \hat{P}(t_{k+1}) \) on the basis \( m_k \) and the motion parameters. These estimates must satisfy (5), (6).
**Prediction** The prediction is obtained by assuming that at the moment $t_{k+1}$ the values of $\phi_i(t_{k+1})$ and $\lambda_i(t_{k+1})$ could have been measured

$$\tilde{\mathbf{X}}(t_{k+1}) = \mathbf{X}(t_k) + VP(t_k)\Delta t$$

$$\tilde{m}_{k+1} = \left( \begin{array}{c} \tilde{m}^\phi_{k+1} \\ \tilde{m}^\lambda_{k+1} \end{array} \right)$$

$$= \left( \begin{array}{c} \dot{Y}(t_{k+1})\cos \phi_i(t_{k+1}) - \dot{X}(t_{k+1})\sin \phi_i(t_{k+1}) \\ \dot{Z}(t_{k+1})\sin \phi_i(t_{k+1})\cos \lambda_i(t_{k+1}) - \dot{Y}(t_{k+1})\sin \lambda_i(t_{k+1}) \end{array} \right).$$

**Correction** After getting $m_{k+1}$ (more precisely the tangents of angles $\phi_i(t_{k+1})$ and $\lambda_i(t_{k+1})$) one can obtain the estimate of the UAV position at the time $t_{k+1}$.

Therefore, the solution of Problem 1 has a form [22]:

$$\mathbf{X}(t_{k+1}) = \mathbf{X}(t_k) + \mathbf{P}(t_k)\Delta t$$

$$\tilde{m}_{k+1} = \left( \begin{array}{c} \tilde{m}^\phi_{k+1} \\ \tilde{m}^\lambda_{k+1} \end{array} \right)$$

$$= \left( \begin{array}{c} \dot{Y}(t_{k+1})\cos \phi_i(t_{k+1}) - \dot{X}(t_{k+1})\sin \phi_i(t_{k+1}) \\ \dot{Z}(t_{k+1})\sin \phi_i(t_{k+1})\cos \lambda_i(t_{k+1}) - \dot{Y}(t_{k+1})\sin \lambda_i(t_{k+1}) \end{array} \right).$$

**Remark 1.** The estimates obtained by (6), (7) are unbiased [17], [21] and give the best linear estimates, they are kept constant until measurement at the time $t_{k+1} > t_k$ and they must be updated by formulas (6), (7) at that moment. Of course, these estimates are not equal to the conditional expectations, but they are projections on the set of preceding measurements $m_1, ..., m_k$, therefore, they are orthogonal to the linear space $L\{m_1, ..., m_k\}$.  

5 Robust filtering on the basis of the UAV motion model

RANSAC method calculates the rotation matrix and the coordinates of the camera $\{A^*, b^*\}$ in the earth coordinate system with some minor error. However, the RANSAC method can give the rudely wrong answer called outlier. For example, it is likely to happen when frames $I_c$ and $I_m$ not depict common objects. Further we provide method which makes a decision about whether a $\{A^*, b^*\}$ is an outlier or not. Here we give the modification of the robust RANSAC [23] based on...
This filter is the extension of the conditionally-optimal Pugachev filter [17] to the case of bearing observations. After the prediction step of Kalman filter we have an estimate of own camera position and the matrix of the mean square errors:

\[
\tilde{X} = \tilde{X}(t_k+1), \quad \tilde{P} = \tilde{P}(t_k+1).
\]

Suppose that the corresponding probability density is Gaussian: since the PKF is the best filter obtained in the form of linear combination of random residuals with almost the same variation one can expect the Gaussian approximation for the resulting error. It gives the opportunity to approximate the probability density distribution by the Gaussian one.

\[
p(X) = \frac{1}{(2\pi)^{1.5}|\tilde{P}|^{0.5}} e^{-\frac{1}{2}(X-\tilde{X})^T\tilde{P}^{-1}(X-\tilde{X})}.
\]

Then pair \(\{A^*, b^*\}\) is considered as an outlier if and only if:

\[
p(b^*) < T,
\]

where \(T\) – manually selected constant threshold. Correction step of the Kalman filter is based on \(\{A^*, b^*\}\) and so in this case this step is passed:

\[
\hat{X}(t_{k+1}) = \tilde{X}(t_{k+1}), \quad \hat{P}(t_{k+1}) = \tilde{P}(t_{k+1}).
\]

### 6 Control of UAV

The problem of the optimal control for system (1) is stochastic one with incomplete information and doesn’t have the explicit solution. However, for practical reasons one can simplify it if to consider the locally optimal control. Here we discuss two problems:

**Problem 2.** Find the locally optimal controls \(\gamma(t_k)\) and \(\theta(t_k)\) at constant velocity \(V\) aimed to keep the motion of UAV along the reference trajectory.

**Problem 3.** Find the locally optimal controls \(\gamma(t_k), \theta(t_k), V(t_k)\) aimed to keep the motion of UAV along the reference trajectory.

#### 6.1 Solution of Problem 2

Assuming that we have some reference trajectory

\[
X_{nom}(t_k) = (X_{nom}(t_k), Y_{nom}(t_k), Z_{nom}(t_k))
\]

we obtain locally optimal controls \(\gamma_c(t_k)\) and \(\theta_c(t_k)\) on the basis of current estimates \(\hat{X}(t_k)\) in order to minimize the deviation in 2-norm from the reference path at the next time \(t_{k+1}\) [19], [20]. Solution is given by formulas (8), (9), where

\[
\tan \theta_c(t_k) = \frac{\Delta Y(t_k) + V \Delta t \cos \gamma(t_k) \sin \theta(t_k)}{\Delta X(t_k) + V \Delta t \cos \gamma(t_k) \cos \theta(t_k)} \tag{8}
\]
and

\[
\tan \gamma_c(t_k) = \frac{\Delta Z(t_k) + V \Delta t \sin \gamma(t_k)}{\cos \theta_c(t_k)(\Delta X(t_k) + V \Delta t \cos \gamma(t_k) \cos \theta(t_k)) + \sin \theta_c(t_k)(\Delta Y(t_k) + V \Delta t \cos \gamma(t_k) \sin \theta(t_k))}.
\]

(9)

\[
\Delta X(t_k) = X_{\text{nom}}(t_k) - \hat{X}_{\text{nom}}(t_k)
\]

6.2 Solution of Problem 3

We determine velocity \(V(t_k)\) and angles \(\gamma_c(t_k), \theta_c(t_k)\) on the basis of the current estimates \(\hat{X}(t_k)\) in order to minimize the deviation from the reference path on the next step, so the solution of the Problem 3 has a form [1], [22]:

\[
V(t_k) = \cos \gamma_c(t_k) \cos \theta_c(t_k) \left( \frac{\Delta X(t_k)}{\Delta t} + V \cos \gamma(t_k) \cos \theta(t_k) \right)
+ \cos \gamma_c(t_k) \sin \theta_c(t_k) \left( \frac{\Delta Y(t_k)}{\Delta t} + V \cos \gamma(t_k) \sin \theta(t_k) \right)
+ \sin \gamma_c(t_k) \left( \frac{\Delta Z(t_k)}{\Delta t} + V \sin \gamma(t_k) \right).
\]

(10)

While angular controls remain the same as above (8), (9).

7 Simulations

Fig. 2 presents one UAV’s flight simulation. The on-board camera oriented in nadir. Blue quadrangle shows the part of the map which is uploaded to the UAV control system memory. Blue dots corresponds to the nominal (desired) trajectory; black dots to the UAV’s position which is affected by wind perturbations and control; red dots mean the estimate of the UAV obtained by pseudomeasurement filter. The aim of control is to lead the real trajectory to the nominal one, however, the estimate of the position on the basis of natural landmarks is possible only when the reference points may be found. That corresponds to square boxes, otherwise there are dots. In total, one can see, how control brings red dot’s trajectory back to blue ones, however, in fact UAV is located on the black trajectory which is not coincide with the blue one. Meanwhile the simulation of flight based on determination of the feature points demonstrates the good performance of the algorithm even for 2D features’ point correspondence [12], see the next Fig. 3.
8 Conclusions

The usage of this sub-optimal filter provides the good potential for image based UAV navigation in GPS denied environment. The angular and velocity controls permit to realize the good tracking in the area of bearing-only measurement. However, we do not aim to demonstrate the possibility of good control on the basis of bearing-only observation. Of course, this type of observations must be used with other measurement systems and our results really open the way to the data fusion. The principal reason for this is that we obtain the unbiased UAV position estimate with known mean square error. Only both of these properties permit to make the data fusion in an optimal way [22]. Here we also suggested an approach to robust RANSAC which takes into account the UAV motion model which is a result of usage the unbiased estimation with known quadratic error. However, another important result had been demonstrated. The usage of the feature points, that are invariant to the scale and the aspect angle, permits to create on-line navigation system based on the observation of underlying surface without the recognition of in advance specified objects. This is the principal advantage of the algorithm, which is extremely important for performance of long-term autonomous UAV missions in dangerous environment.

References

Fig. 3. Scheme of flight based on 2D feature points coordinates


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