Application of the Optical Flow as a Navigation Sensor for UAV ⋆

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Аннотация Recently, relatively small and even micro unmanned aerial vehicles (UAV) came into play and the navigation based on computation of the camera path and the distance to obstacles with the aid of the optical flow (OF) became highly demanded. OF is the field of image motion velocities. The success of the OF implementation is based on the accessibility of its calculation with the aid of relatively simple algorithms, like Lukas-Canade, which admits the simple hardware realization. However, the complete OF is the linear function of linear and angular velocities of the aircraft which permits to use it as an additional sensor of the navigation parameters. This approach to the UAV navigation presumes the on-board camera giving the video sequence of the underlying surface images providing the information about the UAV evolutions. Extraction of the navigation parameters is made on the basis of exact formulas for OF which gives the description of the observation process for estimation based on Kalman filtering. Since the number of the estimating parameters (linear and angular velocities) is substantially less than the number of measurements (practically the number of the camera pixels), one can expect the high accuracy of these parameters estimation.

Keywords: UAV, optical flow, Lukas-Kanade algorithm, filtering.

1 Introduction

Usage of optical devices in remote control of UAV is the natural consequence of the developments in optical-electronic devices and the systems of the data transmission. In the case of remote control the characteristics of such devices must be coordinated with the properties of the human vision, in contrary during the autonomous flight when the optical systems work together with on-board computer servicing for recognition of the objects and determining their position the demands to these devices become different. Roughly speaking the function of the opto-electronic system (OES) + on-board computer is the determining of the camera motion as well as observable objects [11] with the aid of analysis

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of their images. There are two approaches in the usage of OES. The first one is the extraction and determining of the motion of some specific features of the objects by analogy of the human vision, in this case the metrics of such measurements may be easily implemented in the navigation system. The second one is non metrical analysis by analogy of the insects and birds vision [10]. However, recently it appears the possibility to use so-called optical flow (OF) which contains the information about the camera evolution in the implicit form [9]. The problem of the image motion is very urgent in the analysis and optimization of OES for air and space observation system, where non compensated image motion leads to the image degradation [7], [8]. The image motion field is non-uniform across the field of view, but depends linearly on the camera motion velocities [6]. The general methodology for calculation of the image motion field is developed long ago [8], [5] and opens the way to create the navigation sensors. Some examples of the OF usage for light and small UAV may be found in [1]–[4]. However, that are just examples, related to the specific UAV positions, though the calculation of the OF can be used as the general velocity sensor for navigation UAV at arbitrary attitudes. In this work the general procedure for calculation of the OF as a function of the camera (aircraft) motion velocities had been developed and with the algorithms of the OF calculation on the basis of real video sequence it can serve as the additional navigation sensor for UAV velocities. In the future work we are intended to use the measured real OF as a sensor for linear and angular velocities and to incorporate it with inertial-navigation system in autonomous UAV control.

2 Model

The UAV attitude relative to observed surface is determined by its centre of gravity (COG) coordinates and in the surface coordinates (SC) one can write it as: $POS_A(t) = \left(x_A(t), y_A(t), H_A(t)\right)^T$. The angular attitude relatively to SC is determined by: $ORI_A(t) = \left(\alpha_T(t), \alpha_K(t), \alpha_P(t)\right)^T$. Taking into account aircraft common flight heights, one can consider the surface observed by UAV's camera we consider as an infinite plane. Additionally we define the UAV COG coordinates (UC), and Image own coordinates (IC) (2). We use the geometric optics definitions and the pinhole camera model (2). In such case image point coordinates in conjugate focal plane has the following form in IC: $P^{'} = \left(\xi, \eta, -F\right)^T$, where F is the focal distance of camera lens. The general formula for transformation of IMAGE POINT'S coordinates from IC to SC is as follows

$$(\xi^*, \eta^*, z^*) = S_6\left(t\right) S_5\left(t\right) S_4\left(t\right) \left(S_3 S_2 S_1 P^{'} + C\right) + POS_A, \tag{1}$$

where matrix S_1, S_2, S_3 define IC rotation relatively to UC, vector C defines IC translation relatively to UC, matrix S_4, S_5, S_6 define rotation of UC relatively to SC and POS_A defines UAV translation relatively to SC. We assume for simplicity that the camera has zero rotation and translation relatively to UC, thus the lens principal point coincide with UAV's COG. The camera principal optical axis

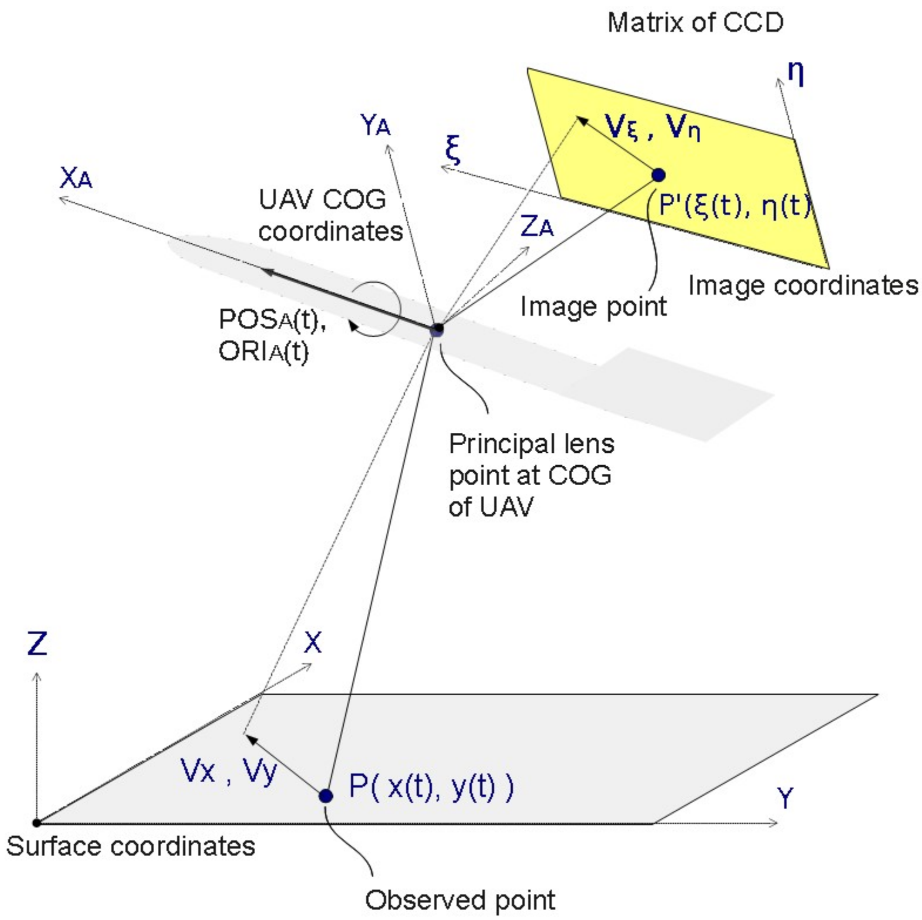

Рис. 1. Surface-UAV-image model

is parallel to UAV's normal vector, i.e. when pitch and bank angles are zeros, the principal optical axis will be orthogonal to surface plane. The camera is rigidly fixed, therefore S_1, S_2, S_3 and C are constant. Assumptions about the camera installation 1 can be simplified using the identity matrix as S_1, S_2, S_3 and zero vector C. The resulting transformation of coordinates will consist of three rotations and one translation: (НАДО ПОДУМАТЬ КАК ПРАВИЛЬНО ЗАПИСАТЬ:)

$$(\xi^*, \eta^*, z^*) = S_6(t)S_5(t)S_4(t)P^{'} + POS_A, \tag{2}$$

where rotation matrix S_4, S_5, S_6 include UAV's orientation angles as a functions of time accordingly to vector $ORI_A(t)$ and z^* is the height of the image point in SC.

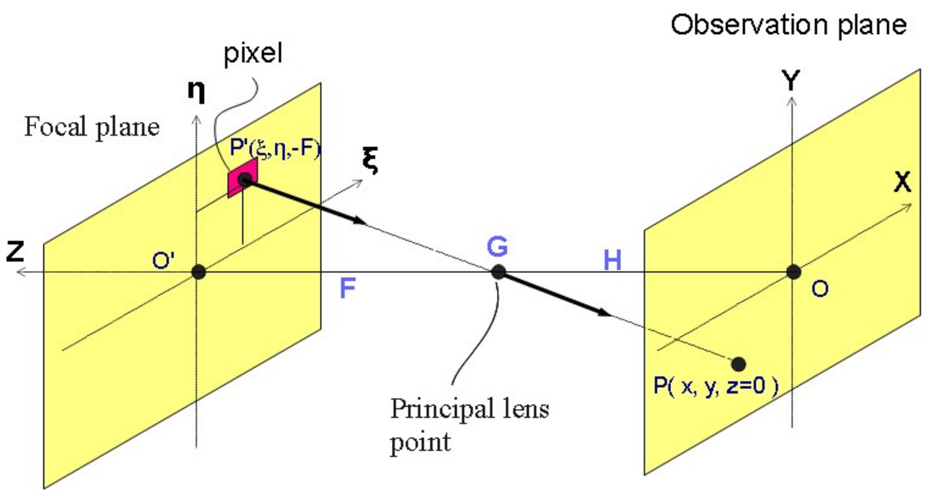

Рис. 2. Pinhole camera model

3 Development of exact optical flow formula

Now, forget about digital camera discrete pixels and assume that we have non discrete image point coordinates $(\xi, \eta, -F)$. From the geometric optics definitions we can derive in analytical form the coordinates of point P of the observed surface visible to image point $P^{'}$ as the intersection point of the optical ray passing through principle lens point G and image point $P^{'}$ in conjugate focal plane with surface plane z=0 (2):

$$
\begin{aligned}
x &= x_A(t) - \frac{H_A(t)}{n}\Bigg\{\Big[\cos(\alpha_P(t))\cos(\alpha_T(t)) + \\
&\quad \sin(\alpha_P(t))\sin(\alpha_K(t))\sin(\alpha_T(t))\Big]\xi + \sin(\alpha_P(t))\cos(\alpha_K(t))\eta - \\
&\quad \Big[-\cos(\alpha_P(t))\sin(\alpha_T(t)) + \sin(\alpha_P(t))\sin(\alpha_K(t))\cos(\alpha_T(t))\Big]F\Bigg\} \\
y &= y_A(t) - \frac{H_A(t)}{n}\Bigg\{\Big[\sin(\alpha_P(t))\cos(\alpha_T(t)) - \\
&\quad \cos(\alpha_P(t))\sin(\alpha_K(t))\sin(\alpha_T(t))\Big]\xi - \cos(\alpha_P(t))\cos\left(\alpha_K(t)\right)\eta - \\
&\quad \Big[-\sin(\alpha_P(t))\sin(\alpha_T(t)) - \cos(\alpha_P(t))\sin(\alpha_K(t))\cos(\alpha_T(t))\Big]F\Bigg\}
\end{aligned}
\tag{3}
$$

where

$$n = \cos(\alpha_K(t))\sin(\alpha_T(t))\xi - \sin(\alpha_K(t))\eta - \cos(\alpha_K(t))\cos(\alpha_T(t))F \tag{4}$$

In general for an image point (ξ, η), we have the analytical form of continuous functions

$$x = x(\xi, \eta, \lambda, t); y = y(\xi, \eta, \lambda, t), \tag{5}$$

where the right-hand side of (5) depends on time-varing parameters:

$$\lambda(t) = (x_A(t), y_A(t), H_A(t), \alpha_T(t), \alpha_K(t), \alpha_P(t), F); \tag{6}$$

Since $P^{'}, P$ are optically conjugate there is a one-to-one relation $(x, y) \leftrightarrow (\xi, \eta)$ which leads to identity:

$$
\begin{aligned}
\xi &= \xi(x(\xi, \eta, \lambda, t), y(\xi, \eta, \lambda, t), \lambda, t); \\
\eta &= \eta(x(\xi, \eta, \lambda, t), y(\xi, \eta, \lambda, t), \lambda, t).
\end{aligned}
\tag{7}
$$

By differentiating both sides of (7) with respect to t we get:

$$
\begin{aligned}
0 &= \frac{\partial \xi}{\partial x}\left[\frac{\partial x}{\partial \xi}\frac{\partial \xi}{\partial t} + \frac{\partial x}{\partial \eta}\frac{\partial \eta}{\partial t} + \frac{\partial x}{\partial t}\right] + \\
&\quad \frac{\partial \xi}{\partial y}\left[\frac{\partial y}{\partial \xi}\frac{\partial \xi}{\partial t} + \frac{\partial y}{\partial \eta}\frac{\partial \eta}{\partial t} + \frac{\partial y}{\partial t}\right] + \frac{\partial \xi}{\partial t} \\
0 &= \frac{\partial \eta}{\partial x}\left[\frac{\partial x}{\partial \xi}\frac{\partial \xi}{\partial t} + \frac{\partial x}{\partial \eta}\frac{\partial \eta}{\partial t} + \frac{\partial x}{\partial t}\right] + \\
&\quad \frac{\partial \eta}{\partial y}\left[\frac{\partial y}{\partial \xi}\frac{\partial \xi}{\partial t} + \frac{\partial y}{\partial \eta}\frac{\partial \eta}{\partial t} + \frac{\partial y}{\partial t}\right] + \frac{\partial \eta}{\partial t}
\end{aligned}
\tag{8}
$$

However,

$$\begin{aligned}
\left[\frac{\partial x}{\partial \xi}\frac{\partial \xi}{\partial t} + \frac{\partial x}{\partial \eta}\frac{\partial \eta}{\partial t} + \frac{\partial x}{\partial t}\right] &= \frac{d}{dt}x(\xi,\eta,\lambda,t) \\
\left[\frac{\partial y}{\partial \xi}\frac{\partial \xi}{\partial t} + \frac{\partial y}{\partial \eta}\frac{\partial \eta}{\partial t} + \frac{\partial y}{\partial t}\right] &= \frac{d}{dt}y(\xi,\eta,\lambda,t),
\end{aligned} \tag{9}$$

so we make substitution into (8) and collect the following matrix form:

$$\begin{pmatrix} \dfrac{\partial \xi}{\partial t} \\ \\ \dfrac{\partial \eta}{\partial t} \end{pmatrix} = - \begin{pmatrix} \dfrac{\partial \xi}{\partial x} & \dfrac{\partial \xi}{\partial y} \\ \\ \dfrac{\partial \eta}{\partial x} & \dfrac{\partial \eta}{\partial y} \end{pmatrix} \begin{pmatrix} \dfrac{dx}{dt} \\ \\ \dfrac{dy}{dt} \end{pmatrix}. \tag{10}$$

From the properties of the Jacobian the further transformation is possible:

$$\begin{pmatrix} \dfrac{\partial \xi}{\partial t} \\ \\ \dfrac{\partial \eta}{\partial t} \end{pmatrix} = - \begin{pmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial x}{\partial \eta} \\ \\ \dfrac{\partial y}{\partial \xi} & \dfrac{\partial y}{\partial \eta} \end{pmatrix}^{-1} \begin{pmatrix} \dfrac{dx}{dt} \\ \\ \dfrac{dy}{dt} \end{pmatrix} \tag{11}$$

4 Determining of the OF by video-sequence

The OF appears as the image pixel translation which leads to the image signal evolution. The signal evolution may be described analytically basing on the following assumptions [9]:

- Continuity of the distribution of the intensity: the intensity changes are caused only by the camera movement relative to observed object;
- Field of translation velocities is continuous;
- Small increments: the sampling frequency is big enough to provide continuous change of the intensity across the image.

When the translation has small increments

$$d\xi(t) = V_\xi(t)dt, \quad d\eta(t) = V_\eta(t)dt$$

changes of the intensity across the image meets the following equation:

$$I(\xi,\eta,t) = I(\xi + d\xi, \eta + d\eta, t + dt),$$

therefore optical flow formula is

$$\frac{\partial I}{\partial t} + \frac{\partial I}{\partial \xi}V_\xi(t) + \frac{\partial I}{\partial \eta}V_\eta(t) = 0. \tag{12}$$

So by observing the sequence of images caused by the camera motion one can evaluate the derivatives and get the equation to determine the OF. In practice the equation (12) is insufficient for the whole field of translation velocities calculation. However with assumptions for the continuity of the intensity and the continuity of the field of translation velocities, this equation is the basis for efficient method for translation velocity components (V_ξ, V_η), calculation known as Lucas-Kanade algorithm [12]. In the algorithm the partial derivatives are estimated in a number of nearby pixels so that assumption for continuity of the field of translation velocities remains true so the translation velocities are computed using least square method.

Doing substitution into the left-hand side of equation (11) the translation velocity components (V_ξ, V_η), then we have the following equation in matrix form for an image point:

$$\begin{pmatrix} V_\xi \\ V_\eta \end{pmatrix} = - \begin{pmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial x}{\partial \eta} \\ \dfrac{\partial y}{\partial \xi} & \dfrac{\partial y}{\partial \eta} \end{pmatrix}^{-1} \begin{pmatrix} \dfrac{dx}{dt} \\ \dfrac{dy}{dt} \end{pmatrix} \Bigg|_{x(\xi,\eta,\lambda,t),\, y(\xi,\eta,\lambda,t)} . \tag{13}$$

On the next step we substitute from (3) $x = x(\xi, \eta, \lambda, t), y = y(\xi, \eta, \lambda, t)$, and derive all required partial derivatives. As the result we have got the linear form for $W_x, W_y, \omega_T, \omega_K, \omega_P, W_H$, which could be written in the following matrix equation:

$$\begin{pmatrix} V_\xi \\ V_\eta \end{pmatrix} = D_1(\xi, \eta, \lambda, t) \begin{pmatrix} W_x \\ W_y \end{pmatrix} + D_2(\xi, \eta, \lambda, t) \begin{pmatrix} \omega_T \\ \omega_K \\ \omega_P \end{pmatrix} + W_H D_3(\xi, \eta, \lambda, t), \tag{14}$$

where (W_x, W_y) is the linear movement velocities of the UAV, $(\omega_T, \omega_K, \omega_P)$ is the rotation velocities relative to UAV's COG, and W_H is the vertical linear movement velocity of the UAV.

5 Usage of the OF for estimation of navigation parameters

The specific view of matrices D_1, D_2, D_3 in the case of the arbitrary attitude is rather cumbersome, however, for the usage of the OF for calculation of velocities one can calculate them in advance and use for least square method of the estimation of linear and angular velocities. Equation 14 open the way to the OF implementation in UAV navigation. For navigation we need to estimate the parameters determining the UAV attitude, that are the linear and angular velocities. If we have a number of the OF observations across the field of view, so the number of left-hand-sides of equation 14, one can estimate the small number of parameters $W_x, W_y, \omega_T, \omega_K, \omega_P, W_H$, say with the aid of technique of least squares. The next step is the estimation of the current values of angles (pitch, yaw, and roll) with the aid of Kalman filter.

5.1 Example

The exact matrix D_1, D_2, D_3 for the case of horizontal flight with constant linear velocity, zero bank and zero pitch is pretty compact to be printed. Making the substitution $H_A(t) = H, \alpha_T(t) = 0, \alpha_K(t) = 0, \alpha_P(t) = 0$ we've got the following matrix:

$$D_1(\xi, \eta, \lambda, t) = \begin{bmatrix} \frac{F}{H} & 0 \\ 0 & \frac{F}{H} \end{bmatrix} \tag{15}$$

$$D_2(\xi, \eta, \lambda, t) = \begin{bmatrix} \frac{\xi\eta}{F} & F + \frac{\xi^2}{F} & -\eta \\ F + \frac{\eta^2}{F} & \frac{\xi\eta}{F} & \xi \end{bmatrix} \tag{16}$$

$$D_3(\xi, \eta, \lambda, t) = \begin{bmatrix} \frac{\xi}{H} \\ \frac{\eta}{H} \end{bmatrix} \tag{17}$$

6 Conclusion

So the methodology of the OF calculation for general UAV attitude had been presented. It permits to use this new tool for navigation of the UAV provided OES and the hardware for on board OF calculation. Since the number of estimating parameters is rather small (just velocities that constitute 6 values) with respect to the number of observations (in general the number of pixels in the frame) including the infromation about the camera (UAV) motion one can expect the high efficiency of this methodology incorporating the optical vision system with INS on the basis of the minimal square method for the current camera (UAV) velocities and the Kalman filter taking into account the dynmaics of the UAV.

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